DIS Robotics Laboratory
Working Paper

Exploration Strategies for General Robotic Systems

Luigi Freda   Giuseppe Oriolo   Francesco Vecchioli
Dipartimento di Informatica e Sistemistica
Università di Roma "La Sapienza"
Via Ariosto 25, 00185 Roma, Italy
{freda,oriolo}@dis.uniroma1.it, francesco_vecchioli@fastwebnet.it

March 19, 2009

Exploration Strategies for General Robotic Systems

Luigi Freda Giuseppe Oriolo Francesco Vecchioli
Dipartimento di Informatica e Sistemistica
Università di Roma "La Sapienza"
Via Ariosto 25, 00185 Roma, Italy
{freda,oriolo}@dis.uniroma1.it, francesco_vecchioli@fastwebnet.it

Abstract

This paper presents a novel method for sensor-based exploration of unknown environments by a general robotic system equipped with multiple sensors. The method is based on the incremental generation of a configuration-space data structure called Sensor-based Exploration Tree (SET). The expansion of the SET is driven by information at the world level, where the perception process takes place. In particular, the frontiers of the explored region efficiently guide the search for informative view configurations. Different exploration strategies may be obtained by instantiating the general SET method with different sampling techniques. Two such strategies are presented and compared by simulations in non-trivial 2D and 3D worlds. A completeness analysis of SET is given in the paper.

I. Introduction

This paper presents a novel exploration method by which a general robotic system equipped with multiple sensors can explore an unknown environment. The method is suitable for generic robotic systems (such as fixed or mobile manipulators, wheeled or legged mobile robots, flying robots), equipped with any number of range finders.

In a *sensor-based exploration*, the robot is required to 'cover' the largest possible part of the world with sensory perceptions. A considerable amount of literature addresses this problem for single-body mobile robots equipped with one sensor, typically an omnidirectional laser range finder. In this context, *frontier-based strategies* [1]–[5] are an interesting class of exploration algorithms. These are based on the idea that the robot should approach the boundary between explored and unexplored areas of the environments in order to maximize the expected utility of robot motions.

The problem of exploring an unknown world using a multi-body robotic system equipped with multiple sensors is more challenging. In fact, the sensing space (the world) and the planning space (the configuration space) are very different in nature: the former is a Euclidean space of dimension 2 or 3, while the latter is a manifold in general with dimension given by the number of configuration coordinates, typically 6 or more. While frontiers at the world level clearly retain their informative value, using this information to efficiently plan actions in configuration space is not straightforward.

In the literature, few works exist that address the sensor-based exploration problem for articulated structures, mainly for fixed-base manipulators equipped with a single sensor, e.g., see [6]–[9]. A related problem is 3D object reconstruction and inspection [10].

The SET (Sensor-based Exploration Tree) method, which was originally presented in [11] for single-sensor robotic systems, is a frontier-based exploration method. The basic idea is to guide the robot so as perform a depth-first exploration of the world, progressively sensing regions that are contiguous from the viewpoint of sensor location. In this process, frontiers are used to efficiently identify informative configurations. The information gathered about the free space is mapped to a configuration space roadmap which is incrementally expanded via a sampling-based procedure. The roadmap is used to select the next view configuration, which is added to the SET. In the exploration process, the robot alternates forwarding/backtracking motions on the SET, which essentially acts as an Ariadne's thread.

This work has been funded by the European Commission's Sixth Framework Programme as part of the project PHRIENDS under grant no. 045359.

In this work, we present *(i)* an extension of the SET method to multi-sensor robotic systems *(ii)* a completeness analysis of the algorithm *(iii)* a SET implementation on non-trivial 2D and 3D worlds. In particular, we discuss how to identify which frontiers are relevant for guiding the perception of each sensor and how to assign priorities to the sensors during view planning.

The paper is organized as follows. The problem setting is given in Sect. II. A general exploration method is outlined in Sect. IV and the SET method is presented in Sect. V. Simulation results in different worlds are reported and discussed in Sect. VII. Some extensions of the present work are mentioned in the concluding section.

II. Problem Setting

The robot wakes up in a unknown world populated by obstacles. Its task is to perform an exploration, i.e. cover the largest possible part of the world with sensory perceptions.

A. Robot and World Models

The robot, denoted by $\mathcal{A}$, is a kinematic chain of r rigid bodies ($r \geq 1$) interconnected by elementary joints. This description includes: fixed-base manipulators, single-body and multiple-body mobile robots, flying robots, humanoids and mobile manipulators.

The *world* $\mathcal{W}$ is a compact connected subset of $I\!R^N$, with $N = 2, 3$. It represents the physical space in which the robot moves and acquires perceptions. $\mathcal{W}$ contains the static obstacles $\mathcal{O}_1, ..., \mathcal{O}_p$, each a compact connected subset of $\mathcal{W}$. One of these obstacles is the world boundary $\partial\mathcal{W}$ which is considered as a 'fence'. Denoting by $\mathcal{O} = \bigcup_{i=1}^m \mathcal{O}_i$ the *obstacle region*, the *free world* is $\mathcal{W}_{\text{free}} = \mathcal{W} \setminus \mathcal{O}$.

The robot *configuration space* has dimension n and is denoted by $\mathcal{C}$, while q is a robot configuration. Let $\mathcal{A}(q)$ be the compact region of $\mathcal{W}$ occupied by the robot at q. The *C-obstacle region* $\mathcal{CO}$ is the set of q such that $\mathcal{A}(q) \cap \mathcal{O} \neq \emptyset$. The *free configuration space* is $\mathcal{C}_{\text{free}} = \mathcal{C} \setminus \mathcal{CO}$.

B. Sensor Model

The robot is equipped with a system of m exteroceptive sensors, whose operation is formalized as follows.

Assuming[1] that the robot is at q, denote by $\mathcal{F}_i(q) \subset I\!R^N$ the compact region occupied by the i-th sensor *field of view*, which is *star-shaped* with respect to the sensor *center* $s_i(q) \in \mathcal{W}$. In $I\!R^2$, for instance, $\mathcal{F}_i(q)$ can be a circular sector with apex $s_i(q)$, opening angle α_i and radius R_i, where the latter is the perception range (see Fig. 1, left). The (total) *sensor field* at q is $\mathcal{F}(q) = \bigcup_{i=1}^m \mathcal{F}_i(q)$.

With the robot at q, a point $p \in \mathcal{W}$ is said to be *visible from the i-th sensor* if $p \in \mathcal{F}_i(q)$ and the open line segment joining p and $s_i(q)$ does not intersect $\partial\mathcal{O} \cup \partial\mathcal{A}(q)$. Denote by $\mathcal{V}_i(q)$ the points of $\mathcal{W}_{\text{free}}$ that are visible from the i-th sensor. At each configuration q, the robot sensory system returns (see Fig. 1, right):

- the *visible free region* (or *view*) $\mathcal{V}(q) = \bigcup_{i=1}^m \mathcal{V}_i(q)$;
- the *visible obstacle boundary* $\mathcal{B}(q) = \partial\mathcal{O} \cap \partial\mathcal{V}(q)$, i.e., all points of $\partial\mathcal{O}$ that are visible from at least one sensor.

The above sensor is an idealization of a 'continuous' range finder. For example, it may be a rotating laser range finder, which returns the distance to the nearest obstacle point along the directions (*rays*) contained in its field of view (with a certain resolution). Another sensory system which satisfies the above description is a stereoscopic camera.

[1]The sensor placement is determined by the robot configuration q. Hence, for each sensor that is not rigidly attached to the robot (e.g., that can independently rotate around a certain axis, or is mounted on a pan-tilt platform), it is necessary to include the corresponding dof's in q.

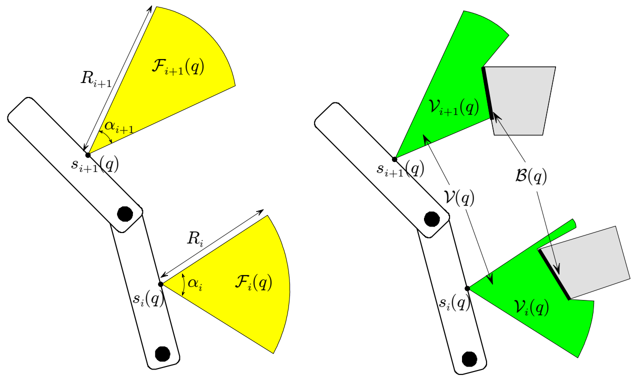

Fig. 1. *Left*: sensor centers $s_i(q)$ and $s_{i+1}(q)$, and the associated fields of view $\mathcal{F}_i(q)$ and $\mathcal{F}_{i+1}(q)$ when the robot is at configuration q. *Right*: The view $\mathcal{V}(q)$ and the visible obstacle boundary $\mathcal{B}(q)$.

C. Exploration task

The robot explores the world through a sequence of view-plan-move actions. Each configuration where a view is acquired is called a *view configuration*. Let q^0 be the initial robot configuration and $q^1, q^2, ..., q^k$ the sequence of view configurations up to the k-th exploration step. When the exploration starts, all the initial robot endogenous knowledge can be expressed as

$$\mathcal{E}^0 = \mathcal{A}(q^0) \cup \mathcal{V}(q^0), \tag{1}$$

where $\mathcal{A}(q^0)$ represents the free volume[2] that the robot body occupies (computed on the basis of proprioceptive sensors) and $\mathcal{V}(q^0)$ is the view at q^0 (provided by the exteroceptive sensors). At step $k \geq 1$, the *explored region* is

$$\mathcal{E}^k = \mathcal{E}^{k-1} \cup \mathcal{V}(q^k).$$

At each step k, $\mathcal{E}^k \subseteq \mathcal{W}_{\text{free}}$ is the current estimate of the free world. Since safe planning requires $\mathcal{A}(q^k) \subset \mathcal{E}^{k-1}$ for any k, we have

$$\mathcal{E}^k = \mathcal{A}(q^0) \cup \left(\bigcup_{i=0}^{k} \mathcal{V}(q^i) \right). \tag{2}$$

A point $p \in \mathcal{W}_{\text{free}}$ is defined *explored* at step k if it is contained in $\mathcal{E}^k$ and *unexplored* otherwise. A configuration q is *safe* at step k if $\mathcal{A}(q) \subset \text{cl}(\mathcal{E}^k)$, where $\text{cl}(\cdot)$ indicates the set closure operation (configurations that bring the robot in contact with obstacles are allowed). The *safe region* $\mathcal{S}^k \subseteq \mathcal{C}_{\text{free}}$ collects the configurations that are safe at step k, and represents a configuration space image of $\mathcal{E}^k$. A path in $\mathcal{C}$ is *safe* at step k if it is completely contained in $\mathcal{S}^k$.

The goal of the exploration is to expand $\mathcal{E}^k$ as much as possible as k increases. Assume that the robot can associate an information gain $I(q,k)$ to any (safe) q at step k. This is an estimate of the world information which can be discovered at the current step by acquiring a view from q.

Consider the k-th exploration step, which starts with the robot at q^k. Let $\mathcal{Q}^k \subset \mathcal{S}^k$ be the *informative safe region*, i.e. the set of configurations which *(i)* have non-zero information gain, and *(ii)* can be reached[3] from q^k through a path that is safe at step k. The exploration can be considered *completed* at step k if $\mathcal{Q}^k = \emptyset$, i.e., no informative configuration can be safely reached.

[2]Often, $\mathcal{A}(q^0)$ in (1) is replaced by a larger free volume $\widetilde{\mathcal{A}}$ whose knowledge comes from an external source. This may be essential for planning safe motions in the early stages of an exploration.

[3]The reachability requirement accounts for possible kinematic constraints to which robot may be subject.

D. Information Gain

Throughout this paper we assume the following definition of information gain.

At step k, the boundary of the explored region $\partial\mathcal{E}^k$ is the union of two *disjoint* sets:

- the *obstacle boundary* $\partial\mathcal{E}^k_{\text{obs}}$, i.e., the part of $\partial\mathcal{E}$ which consists of detected obstacle surfaces;
- the *free boundary* $\partial\mathcal{E}^k_{\text{free}}$, i.e., the complement of $\partial\mathcal{E}^k_{\text{obs}}$, which leads to potentially explorable areas.

One has $\partial\mathcal{E}^k_{\text{obs}} = \bigcup_{i=0}^{k} \mathcal{B}(q^i)$ and $\partial\mathcal{E}^k_{\text{free}} = \partial\mathcal{E}^k \setminus \partial\mathcal{E}^k_{\text{obs}}$.

Let $\mathcal{V}(q,k)$ be the *simulated view*, i.e., the view which would be acquired from q if the obstacle boundary were $\partial\mathcal{E}^k_{\text{obs}}$. The information gain $I(q,k)$ is defined as the measure of the set of unexplored points lying in $\mathcal{V}(q,k)$ [3], [7]. The SET method also makes use of the partial versions of these concepts, denoted respectively by $\mathcal{V}_j(q,k)$ and $I_j(q,k)$, which only consider the contribution of the j-th sensor. While $\mathcal{V}(q,k) = \bigcup_{j=1}^{m} \mathcal{V}_j(q,k)$, it is $I(q,k) \neq \sum_{j=1}^{m} I_j(q,k)$, since partial simulated views may overlap.

Finally, let $\mathcal{Q}^k_j = \{q \in \mathcal{Q}^k \,|\, I_j(q,k) \neq 0\}$ be the *partial informative safe region* of the j-th sensor. It is $\mathcal{Q}^k = \bigcup_{j=1}^{m} \mathcal{Q}^k_j$.

III. Explorable Regions and View Coverage Problems

The defined exploration task belongs to the more general class of view coverage problems, which is presented in this section. The following discussion introduces some conditions for the existence of a solution to the exploration problem. Moreover, it presents some cases in which these condition are not satisfied and, thereby, the exploration problem has no solution.

A. Reachable Region

At step k, let $\mathcal{R}^k \subset \mathcal{S}^k$ be the *reachable region*, i.e., the set of configurations which can be reached from q^k through a path that is safe at step k. The reachable region has the following properties:

- it is a connected region of the configuration space containing q^k;
- $\mathcal{Q}^k \subset \mathcal{R}^k \subset \mathcal{S}^k$.

Note that the informative safe region $\mathcal{Q}^k$ is not connected in general.

B. Maps and Important Regions

In this section, we introduce three maps which can be conveniently used to restate the previous definitions in a functional form. These better explicit the dependances between sets in world and in the configuration space.

The *safe region map*, denoted by

$$m_s : \text{pow}(\mathcal{W}) \to \text{pow}(\mathcal{C}),$$

associates each set $\mathcal{E} \subseteq \mathcal{W}$ to the set $m_s(\mathcal{E}) = \{q \in \mathcal{C} \,|\, \mathcal{A}(q) \subseteq \text{cl}(\mathcal{E})\}$. It is $\mathcal{S}^k = m_s(\mathcal{E}^k)$.

The *reachable region map*, denoted by

$$m_r : \mathcal{C} \times \text{pow}(\mathcal{W}) \to \text{pow}(\mathcal{C}),$$

associates each pair $(q,\mathcal{E}) \in \mathcal{C} \times \text{pow}(\mathcal{W})$ to the set $m_r(q,\mathcal{E})$ which is the set of configurations of $m_s(\mathcal{E})$ which are reachable[4] from q. It is $\mathcal{R}^k = m_r(q^k,\mathcal{E}^k) = m_r(q^0,\mathcal{E}^k)$ since $q^k \in \mathcal{R}^{k-1} = m_r(q^0,\mathcal{E}^{k-1})$ for any $k \geq 1$.

It is easy to show that

$$\mathcal{E}' \subseteq \mathcal{E} \;\Rightarrow\; m_s(\mathcal{E}') \subseteq m_s(\mathcal{E}) \;\wedge\; m_r(\mathcal{E}') \subseteq m_r(\mathcal{E}). \tag{3}$$

[4]The reachability requirement accounts for possible kinematic/dynamic constraints to which robot may be subject.

Furthermore, $m_s(\mathcal{E}') = m_s(\mathcal{E})$ (or $m_r(\mathcal{E}') = m_r(\mathcal{E})$) does not imply $\mathcal{E}' = \mathcal{E}$. Clearly, since it is

$$\mathcal{E}^0 \subseteq \mathcal{E}^1 \subset ... \subseteq \mathcal{E}^k$$

one has

$$\mathcal{S}^0 \subseteq \mathcal{S}^1 \subset ... \subseteq \mathcal{S}^k = m_s(\mathcal{E}^k), \qquad \mathcal{R}^0 \subseteq \mathcal{R}^1 \subset ... \subseteq \mathcal{R}^k = m_r(q^0, \mathcal{E}^k)$$

Table I presents and compares important regions and their on-line estimates using the above maps.

Important regions	On-line estimates
$\mathcal{W}_{\text{free}}$	$\mathcal{E}^k$
$\mathcal{C}_{\text{free}} \quad = m_s(\mathcal{W}_{\text{free}})$	$\mathcal{S}^k \quad = m_s(\mathcal{E}^k)$
$\mathcal{C}_{\text{con}} \quad = m_r(q^0, \mathcal{W}_{\text{free}})$	$\mathcal{R}^k \quad = m_r(q^0, \mathcal{E}^k)$

TABLE I

The *explorable region map*, denoted by

$$m_e : \text{pow}(\mathcal{C}) \to \mathcal{W}$$

associates each set $\mathcal{R} \subset \mathcal{C}$ to the set $m_e(\mathcal{R}) = \bigcup_{q \in \mathcal{R}} \mathcal{V}(q)$. Clearly, it is

$$\mathcal{R}' \subseteq \mathcal{R} \;\Rightarrow\; m_e(\mathcal{R}') \subseteq m_e(\mathcal{R}). \tag{4}$$

In general, the reverse does not hold.

C. Maximum Explorable Regions and Maximum Coverable Region

The ‘initial condition’ for a robot exploration is the pair $(q^0, \mathcal{E}^0) \in \mathcal{C}_{\text{free}} \times \text{pow}(\mathcal{W}_{\text{free}})$ *(exploration initial condition*). Starting from it, any exploration method generates an *exploration sequence*, i.e., a sequence of view configurations $\{q^i\}$ such that $q^i \in \mathcal{R}^{i-1} = m_r(q^0, \mathcal{E}^{i-1})$ for any i.

At step k, let $\mathcal{E}^k_{\text{max}} \subseteq \mathcal{W}$ be the *maximum explorable region*, i.e., the set

$$\mathcal{E}^k_{\text{max}} = \mathcal{E}^0 \cup \left[\bigcup_{q \in m_r(q^0, \mathcal{E}^{k-1}_{\text{max}})} \mathcal{V}(q) \right]$$

where $\mathcal{E}^0_{\text{max}} = \mathcal{E}^0$. It is

$$\mathcal{E}^k_{\text{max}} = \mathcal{E}^0 \cup [m_e \circ m_r(q^0, \mathcal{E}^{k-1}_{\text{max}})] = \mathcal{E}^0 \cup [(m_e \circ m_r)^k(q^0, \mathcal{E}^0)]$$

where $(m_e \circ m_r)^k$ denotes (with abuse of notation) the composition of $m_e \circ m_r$ for k times. Clearly, the set $\mathcal{E}^k_{\text{max}}$ depends on the initial condition $(q^0, \mathcal{E}^0)$ and determines an un upper-bound for any explored region which can be obtained at step k starting from the same initial condition, i.e.,

$$\mathcal{E}^k \subseteq \mathcal{E}^k_{\text{max}} \tag{5}$$

Let $\mathcal{E}_{\text{cov}} \subseteq \mathcal{W}$ be the *maximum coverable region*, i.e., the set

$$\mathcal{E}_{\text{cov}} = \mathcal{E}^0 \cup \left[\bigcup_{q \in \mathcal{C}_{\text{con}}} \mathcal{V}(q) \right].$$

where $\mathcal{C}_{\text{con}} = m_r(q^0, \mathcal{W}_{\text{free}})$. One has

$$\mathcal{E}_{\text{cov}} = \mathcal{E}^0 \cup m_e(\mathcal{C}_{\text{con}}) = \mathcal{E}^0 \cup [m_e \circ m_r(q^0, \mathcal{W}_{\text{free}})].$$

Indeed, the maximum coverable region $\mathcal{E}_{\text{cov}}$ is the largest region of the world which can be explored by the robot starting from $(q^0, \mathcal{E}^0)$.

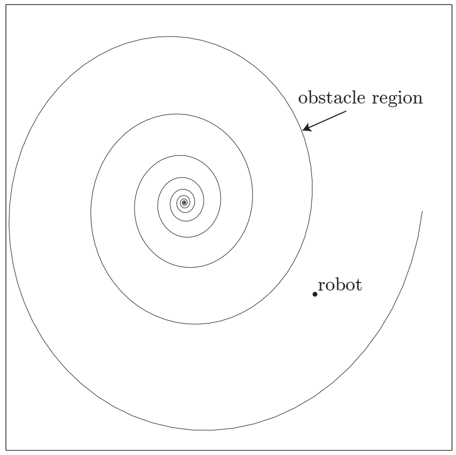

Fig. 2.

Proposition 3.1: Given an initial condition $(q^0, \mathcal{E}^0)$, it is at each step k

$$\mathcal{E}^k \subseteq \mathcal{E}^k_{\max} \subseteq \mathcal{E}_{\mathrm{cov}} \tag{6}$$

Proof: Since it is $\mathcal{E}^{k-1}_{\max} \subseteq \mathcal{W}_{\mathrm{free}}$ for any $k \geq 1$, one has $m_r(q^0, \mathcal{E}^{k-1}_{\max}) \subseteq m_r(q^0, \mathcal{W}_{\mathrm{free}})$. Applying the map m_e to this last relation and recalling eq. (4), the thesis easily follows. ■

Proposition 3.2: The following implication holds:

$$\mathcal{Q}^k = \emptyset \quad \Rightarrow \quad \mathcal{E}^k = \mathcal{E}^k_{\max} = \mathcal{E}^{k+1}_{\max}$$

Proof: It is

$$\mathcal{Q}^k = \emptyset \quad \Rightarrow \quad \mathcal{E}^k = \mathcal{E}^0 \cup \left[\bigcup_{q \in \mathcal{R}^k} \mathcal{V}(q) \right]. \tag{7}$$

In fact, if $\mathcal{Q}^k = \emptyset$ there is no $q \in \mathcal{R}^k$ such that $\mathcal{V}(q,k) \setminus \mathcal{E}^k \neq \emptyset$. This in turn implies that there is no $q \in \mathcal{R}^k$ such that $\mathcal{V}(q) \setminus \mathcal{E}^k \neq \emptyset$ (note that $\mathcal{V}(q) \subset \mathcal{V}(q,k)$ since the obstacle boundary $\partial \mathcal{E}^k_{\mathrm{obs}}$ is contained in the obstacle region $\mathcal{O}$). Hence, eq. (7) follows.

At this point, note that there exists $j \in I\!N$ such that $m_r(q^0, \mathcal{E}^j_{\max}) \subseteq \mathcal{R}^k$ (at least, it is $j = 0$ since $m_r(q^0, \mathcal{E}^0) = \mathcal{R}^0 \subseteq \mathcal{R}^k$, where $\mathcal{E}^0 = \mathcal{E}^0_{\max}$). This implies

$$\mathcal{E}^{j+1}_{\max} = \mathcal{E}^0 \cup \left[\bigcup_{q \in m_r(q^0, \mathcal{E}^j_{\max})} \mathcal{V}(q) \right] \subseteq \mathcal{E}^k = \mathcal{E}^0 \cup \left[\bigcup_{q \in \mathcal{R}^k} \mathcal{V}(q) \right].$$

Thereby, it is $m_r(q^0, \mathcal{E}^{j+1}_{\max}) \subseteq m_r(q^0, \mathcal{E}^k) = \mathcal{R}^k$ which in turn implies $\mathcal{E}^{j+2}_{\max} \subseteq \mathcal{E}^k$. By induction, one has $\mathcal{E}^k_{\max} \subseteq \mathcal{E}^{k+1}_{\max} \subseteq \mathcal{E}^k$. Recalling eq. (5), the thesis follows. ■

Corollary 3.3: Given an initial condition $(q^0, \mathcal{E}^0)$, a necessary condition for the existence of a finite exploration sequence of length $k < \infty$ such that $\mathcal{Q}^k = \emptyset$ is:

$$\exists l \in I\!N \; : \; \mathcal{E}^l_{\max} = \mathcal{E}^{l+1}_{\max}, \tag{8}$$

where, in general, $l \leq k$.

Remark 3.4: In general, $\mathcal{E}^l_{\max} = \mathcal{E}^{l+1}_{\max}$ does not imply $\mathcal{E}^l_{\max} = \mathcal{E}_{\mathrm{cov}}$ since the initial geometric restrictions imposed by $\mathcal{E}^0$ may eventually hamper the exploration of some portions of $\mathcal{E}_{\mathrm{cov}}$.

Remark 3.5: There are cases in which for any $(q^0, \mathcal{E}^0)$ there is no $l \in I\!N$ such that $\mathcal{E}^l_{\max} = \mathcal{E}^{l+1}_{\max}$. For instance, consider the planar case shown in Fig. 2: the robot is a point equipped with an omnidirectional

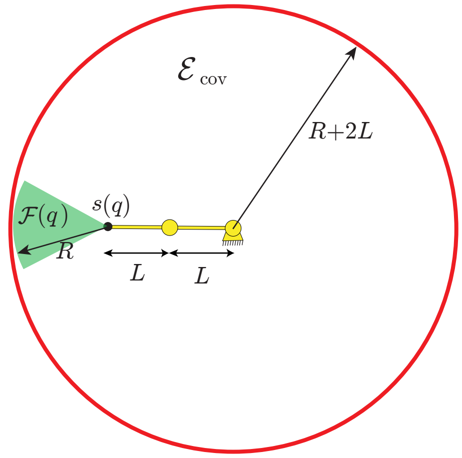

Fig. 3.

rangefinder; the world $\mathcal{W}$ is a squared box and the obstacle $\mathcal{O}$ region is a logarithmic spiral. In this case, for any q^0, the set $\mathcal{E}_{\rm cov}$ is bounded and equal to $\mathcal{W}_{\rm free}$. However, for any $(q^0, \mathcal{E}^0)$ the sequence $\{\mathcal{E}^i_{\rm max}\}$ is strictly increasing, i.e., $\mathcal{E}^i_{\rm max} \subset \mathcal{E}^{i+1}_{\rm max}$ for any $i \in I\!N$.

Remark 3.6: Condition (8) is only a necessary condition for the existence of a finite exploration sequence of length k such that $\mathcal{Q}^k = \emptyset$. Consider the case depicted in Fig. 3: the robot is a planar 2R manipulator with a fixed base, two links of length L and a single rangefinder mounted on the tip of the second link. No obstacles are present in the world. Here, the maximum coverable region $\mathcal{E}_{\rm cov}$ is the closed (red) circle of radius $R+2L$ centered at the base joint. If $\mathcal{E}^0$ is a circle of radius $2L$ centered at the base joint and $q^0 \in m_r(q^0, \mathcal{E}^0)$, one has[5] $\mathcal{E}^1_{\rm max} = \mathcal{E}^2_{\rm max} = \mathcal{E}_{\rm cov}$. Nevertheless, there is no finite exploration sequence of length $k < \infty$ such that $\mathcal{Q}^k = \emptyset$. Intuitively, this is due to the fact that any robot sensor view can at most 'touch' $\partial\mathcal{E}_{\rm cov}$ in a single point, i.e. for any pair $(p', q') \in \partial\mathcal{E}_{\rm cov} \times \mathcal{C}_{\rm con}$ such that $p' \in \mathcal{V}(q')$, one has $p' = \mathcal{V}(q') \cap \partial\mathcal{E}_{\rm cov}$. This implies that in order to complete the exploration the robot needs to acquire at least ∞^1 views in order to cover $\partial\mathcal{E}_{\rm cov} \subset \mathcal{E}_{\rm cov}$ (in this example $\mathcal{E}_{\rm cov}$ is a closed set, this is not true in general). This is due to the fact that the radius of curvature of the sensor field of view boundary (i.e. the perception range R) is smaller than the radius of curvature of the maximum coverable region boundary (i.e. $R + 2L$).

Remark 3.7: The example described in the previous Remark 3.6 can be generalized in order to identify more general 'pathological' cases, i.e., cases in which there is no finite exploration sequence of length $k < \infty$ such that $\mathcal{Q}^k = \emptyset$.

Assume there exists $l \in I\!N$ such that $\mathcal{E}^l_{\rm max} = \mathcal{E}^{l+1}_{\rm max}$. Let $\mathcal{K}$ be the *views boundary*, i.e. the set

$$\mathcal{K} = \left[\bigcup_{q \in m_r(q^0, \mathcal{E}^l_{\rm max})} \partial\mathcal{V}(q) \right] \setminus \left[\bigcup_{q \in m_r(q^0, \mathcal{E}^l_{\rm max})} \overset{\circ}{\mathcal{V}}(q) \right].$$

where $\overset{\circ}{\mathcal{V}}(q)$ denotes the interior of $\mathcal{V}(q)$. In general, $\mathcal{K}$ can have a non-empty intersection with the interior of $\mathcal{E}^l_{\rm max}$ (see the example shown in Fig. 4).

Assume there exists a manifold $\mathcal{M} \subset \mathcal{K}$ with the following property: for any pair $(p', q') \in \mathcal{M} \times m_r(q^0, \mathcal{E}^l_{\rm max})$ such that $p' \in \mathcal{V}(q')$ it is $p' = \mathcal{M} \cap \mathcal{V}(q')$ (i.e., any sensor view can at most 'touch' the manifold $\mathcal{M}$ in a single point). Note that a necessary condition for the set $\mathcal{M}$ to satisfy this last property

[5] In this simple example, it is easy to envisage other pairs $(q^0, \mathcal{E}^0)$ for which $\exists l \in I\!N : \mathcal{E}^l_{\rm max} = \mathcal{E}^{l+1}_{\rm max} = \mathcal{E}_{\rm cov}$.

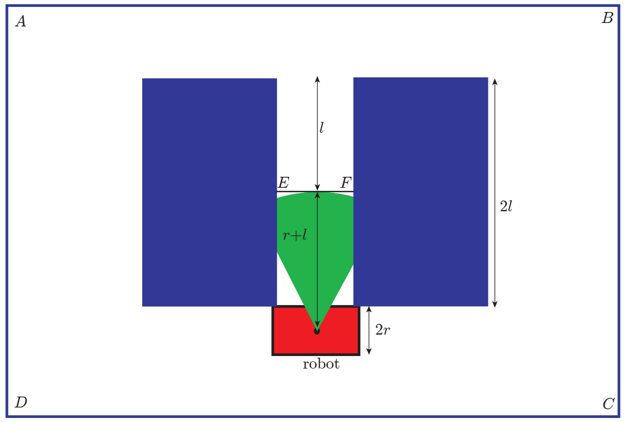

Fig. 4. The world, the obstacles and the robot are rectangles in $I\!R^2$. In particular, the robot is the depicted small rectangle (red/thick line) and is only allowed to translate. $\mathcal{E}_{\text{cov}}$ equals $\mathcal{W}_{\text{free}}$. Given the particular sensor perception range and the sizes of the obstacles and the robot, the views boundary $\mathcal{K}$ is the union of the world boundary and the open line segment $EF \subset \mathcal{E}_{\text{cov}}$.

is

$$\mathcal{M} \cap \left[\bigcup_{q \in m_r(q^0, \mathcal{E}^l_{\max})} \overset{\circ}{\mathcal{V}}(q) \right] = \emptyset.$$

In this case, as in the previous example, the robot needs to acquire an uncountable number of views in order to completely cover $\mathcal{M} \subset \mathcal{E}^l_{\max}$. Hence, there is no finite exploration sequence of length $k < \infty$ such that $\mathcal{Q}^k = \emptyset$

D. Extension and Dimension of the Explorable Regions

Given a set $\mathcal{E} \subseteq \mathcal{W}$ and a pair of points $x, y \in \mathcal{E}$, denote by $\text{ssp}(x, y)$ the shortest path which joins x and y and is contained in $\text{cl}(\mathcal{E})$ (*shortest safe path*). Denote by $\text{length}(\tau) \in I\!R$ the length of a path τ. Denote by $\text{sd}(\mathcal{E})$ be the *safe diameter* of the set $\mathcal{E}$, i.e.,

$$\sup\{\text{length}(\text{ssp}(x, y)) \,:\, (x, y) \in \mathcal{E}\}.$$

Clearly, the diameter of a set $\mathcal{E}$, defined as $\sup\{\|x - y\| \,:\, (x, y) \in \mathcal{E}\}$, is always smaller than its safe diameter.

Proposition 3.8: Given an initial condition $(q^0, \mathcal{E}^0)$, it is

$$\text{sd}(\mathcal{E}^k_{\max}) \leq \text{sd}(\mathcal{E}^0) + 2kR \tag{9}$$

Proof: For any pair of points $x, y \in \mathcal{E}^k_{\max}$, one of the following three cases occurs:

- both points $x, y \in \mathcal{E}^{k-1}_{\max}$. Hence $\text{length}(\text{ssp}(x, y)) \leq \text{sd}(\mathcal{E}^{k-1}_{\max})$.
- both points $x, y \in \mathcal{E}^k_{\max} \setminus \mathcal{E}^{k-1}_{\max}$. In this case, there are two configurations $q_x, q_y \in m_r(q^0, \mathcal{E}^{k-1}_{\max})$ such that $x \in \mathcal{V}(q_x)$, $s(q_x) \in \mathcal{E}^{k-1}$, $y \in \mathcal{V}(q_y)$ and $s(q_y) \in \mathcal{E}^{k-1}$. Hence, a safe path joining x and y can be built connecting: the segment $(x, s(q_x)) \subset \mathcal{V}(q_x)$, a safe path joining $s(q_x)$ and $s(q_y)$ which is contained in $\mathcal{E}^{k-1}_{\max}$, the segment $(y, s(q_y)) \subset \mathcal{V}(q_y)$. It follows $\text{length}(\text{ssp}(x, y)) \leq \text{sd}(\mathcal{E}^{k-1}_{\max}) + 2R$.
- only one of the two points belongs to $\mathcal{E}^k_{\max} \setminus \mathcal{E}^{k-1}_{\max}$. Using the same arguments, it follows $\text{length}(\text{ssp}(x, y)) \leq \text{sd}(\mathcal{E}^{k-1}_{\max}) + R$.

Consequently, one has:

$$\text{sd}(\mathcal{E}^k_{\max}) \leq \text{sd}(\mathcal{E}^{k-1}_{\max}) + 2R$$

and the thesis easily follows. ∎

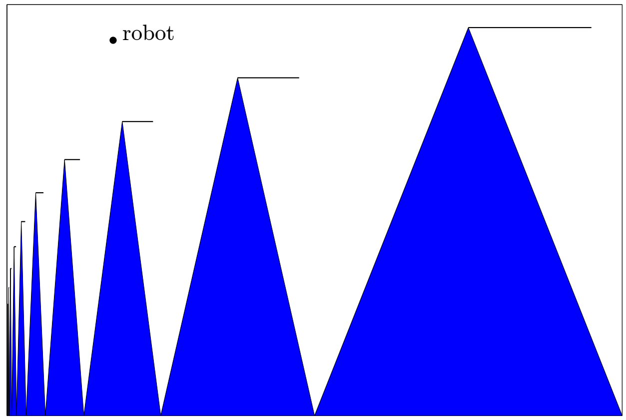

Fig. 5. A point robot moves in a rectangular box in which the obstacles are an infinite sequence of triangles and line segments whose dimensions exponentially decrease.

Using similar arguments, it is easy to infer that

$$\mathrm{sd}(\bigcup_{i=0}^{k} \mathcal{V}(q^i)) \leq 2(k+1)R \tag{10}$$

Remark 3.9: From (9), it follows that given an initial pair $(q^0, \mathcal{E}^0)$, where $\mathcal{E}^0$ has a finite safe diameter, if there is $l \in I\!N$ such that $\mathcal{E}^l_{\max} = \mathcal{E}^{l+1}_{\max}$, then the final maximum explorable region $\mathcal{E}^l_{\max}$ has a finite safe diameter.

Corollary 3.10: Given an initial condition $(q^0, \mathcal{E}^0)$ such that $\mathrm{sd}(\mathcal{E}^0) < \infty$, a necessary condition for the existence of an integer $l \in I\!N$ such that

$$\mathcal{E}^l_{\max} = \mathcal{E}^{l+1}_{\max} = \mathcal{E}_{\mathrm{cov}}$$

is that $\mathrm{sd}(\mathcal{E}_{\mathrm{cov}}) < \infty$.

According to this corollary, in the case of Fig. 2, $\mathcal{E}_{\mathrm{cov}}$ is bounded but has not a finite safe diameter.

It is worth noting that the above condition is only necessary. For instance, consider a point robot equipped with an omnidirectional rangefinder in the rectangular world depicted in Fig. 5. Here, the obstacles are an infinite sequence of triangles and line segments (on the bottom) whose dimensions exponentially decrease. In this case, for any q^0, the set $\mathcal{E}_{\mathrm{cov}}$ equals $\mathcal{W}_{\mathrm{free}}$. However, for any $(q^0, \mathcal{E}^0)$ the sequence $\{\mathcal{E}^i_{\max}\}$ is strictly increasing, i.e., $\mathcal{E}^i_{\max} \subset \mathcal{E}^{i+1}_{\max}$ for any $i \in I\!N$.

Remark 3.11: In the worlds depicted in Fig. 2 and Fig. 5, some portions of the obstacle boundaries have fractal characteristics. That means they have some kind of 'recursive' definition, a fine structure at arbitrarily small scales, self-similarity and this implies that the Hausdorff dimension of the world is greater than its topological dimension.

E. View Coverage Problems

Definition 3.12: Off-line view coverage problem (generalized art-gallery problem).
Given a starting configuration $q^0 \in \mathcal{C}_{\mathrm{free}}$, find a finite collection of view configurations $\{q^0, q^1, ..., q^k\} \subset \mathcal{C}_{\mathrm{con}} = m_r(q^0, \mathcal{W}_{\mathrm{free}})$ such that:

(i) $\bigcup_{i=0}^{k} \mathcal{V}(q^i) = m_e(\mathcal{C}_{\mathrm{con}})$,

(ii) k is minimum

or correctly report that such a collection of view configurations does not exist.

According to eq. 10 a necessary condition for the existence of a solution to the off-line view coverage problem is that $\mathrm{sd}(m_e(\mathcal{C}_{\text{con}})) < \infty$. Note that, in the off-line view coverage problem, no explicit restrictions are imposed on the collection of view configurations (e.g. they can be an exploration sequence or not). The standard art-gallery problem with a point robot and a polygonal world is a an example of off-line view coverage problem.

Definition 3.13: On-line view coverage problem (exploration problem).
Given an initial pair $(q^0, \mathcal{E}^0) \in \mathcal{C}_{\text{free}} \times \mathrm{pow}(\mathcal{W}_{\text{free}})$ such that $\mathcal{A}(q^0) \subseteq \mathcal{E}^0$, find a finite exploration sequence $\{q^0, q^1, ..., q^k\}$ such that

(i) $\mathcal{Q}^k = \emptyset$,

(ii) k is minimum.

or correctly report that such an exploration sequence does not exist.

According to Corollary 3.3, a necessary condition for the existence of a solution to the above exploration problem is that

$$\exists l \in I\!N \ : \ \mathcal{E}^l_{\max} = \mathcal{E}^{l+1}_{\max}.$$

Given an (exploration sequence) solution of length k, it is $\mathcal{E}^k = \mathcal{E}^l_{\max}$ with $k \geq l$.

A *complete exploration algorithm* is capable of solving any instance of the exploration problem.

F. Pathological Cases

A *pathological case* is characterized by the fact that for any $(q^0, \mathcal{E}^0)$ there is no finite exploration sequence of length $k < \infty$ such that $\mathcal{Q}^k = \emptyset$. That means, that any exploration sequence $\{q^i\}$ which can be obtained maximizing $I(q, i)$ over $\mathcal{Q}^i$ at each step i, corresponds to an infinite sequence $\{I(q^i, i)\} > 0$ which at most tends to zero. Hence, in a pathological case, $\mathcal{Q}^i$ never becomes empty.

There are two main class of pathological cases:

1) the first class consists in all the exploration problem instances for which there is no finite $l \in I\!N$ such that $\mathcal{E}^l_{\max} = \mathcal{E}^{l+1}_{\max}$. The cases depicted in Figs. 2 and 5 belong to such class. Here, the pathology arises from the arduous world geometric characteristics (*world pathology*).
2) The second class collects all the remaining cases. The cases depicted in Fig. 3 and 4 belongs to this second class. Here, in most cases, the pathology is caused by the 'sensor limits' and the discrete nature of view sensing. Indeed, these two factors can issue the problematic situation presented in remark 3.7 (*sensor pathology*). In such cases, if the robot were ideally allowed to perform a continuous view sensing, the pathology would be removed and, thereby, the exploration could be completed.

IV. Exploration Methods

A general exploration method (Fig. 6) searches for the next view configuration in $\mathcal{Q}^k \cap \mathcal{D}(q^k, k)$, where $\mathcal{D}(q^k, k) \subseteq \mathcal{C}$ is a compact *admissible set* around q^k at step k, whose size determines the scope of the search. For example, if $\mathcal{D}(q^k, k) = \mathcal{C}$, a global search is performed, whereas the search is local if $\mathcal{D}(q^k, k)$ is a neighborhood of q^k.

If $\mathcal{Q}^k \cap \mathcal{D}(q^k, k)$ is not empty, q^{k+1} is selected in it according to some criterion (e.g., information gain maximization). The robot then moves to q^{k+1} to acquire a new view (*forwarding*). Otherwise, the robot returns to a previously visited q^b ($b < k$) such that $\mathcal{Q}^k \cap \mathcal{D}(q^b, k)$ is not empty (*backtracking*). Given that the world is static, it is not necessary to acquire again a view from q_b. Hence, the actual number of views gathered so far may be less than k.

To specify an exploration method, one must define:

- an information gain;
- a forwarding selection strategy;
- an admissible set $\mathcal{D}(q^k, k)$;

GENERAL EXPLORATION METHOD

if $\mathcal{Q}^k \cap \mathcal{D}(q^k, k) \neq \emptyset$ *%forwarding%*

choose new q^{k+1} in $\mathcal{Q}^k \cap \mathcal{D}(q^k, k)$

move to q^{k+1} and acquire sensor view

else *%backtracking%*

choose visited q^b $(b < k)$ such that $\mathcal{Q}^k \cap \mathcal{D}(q^b, k) \neq \emptyset$

move to q^b

Fig. 6. The k-th step of a general exploration method.

- a backtracking selection strategy.

Due the complexity associated to the computation of $\mathcal{S}^k$, an efficient procedure to predict whether $\mathcal{Q}^k \cap \mathcal{D}(q^k, k)$ is non-empty or not would be useful.

A. A necessary condition for a complete exploration strategy

On the whole a general exploration method selects the next view configuration in the set

$$\bigcup_{j=0}^{k} \mathcal{Q}^k \cap D(q^j, k) = \mathcal{Q}^k \cap \bigcup_{j=0}^{k} D(q^j, k).$$

The set $\bigcup_{j=0}^{k} D(q^j, k)$ is referred to as the *total admissible set* at step k. The exploration strategy ends when the set $\mathcal{Q}^k \cap \bigcup_{j=0}^{k} D(q^j, k)$ is empty (*termination condition*), i.e., when the total admissible set does not contain any more informative configuration.

Recall that an exploration is completed at step k if $\mathcal{Q}^k = \emptyset$. Hence, an exploration strategy can be considered *complete* if for any exploration problem instance for which a solution exists, it generates a finite exploration sequence $q^0, q^1, q^2, ..., q^l$ such that $\mathcal{Q}^l = \emptyset$ in finite time.

A necessary condition for the completeness of the general exploration strategy described in Sect. IV is that the following implication holds at each step k:

$$\mathcal{Q}^k \cap \bigcup_{j=0}^{k} D(q^j, k) = \emptyset \quad \Rightarrow \quad \mathcal{Q}^k = \emptyset. \tag{11}$$

i.e., the termination condition must occur only when the exploration is actually completed. Equivalently, it must be at each exploration step k

$$\mathcal{Q}^k \neq \emptyset \quad \Rightarrow \quad \mathcal{Q}^k \cap \bigcup_{j=0}^{k} D(q^j, k) \neq \emptyset \tag{12}$$

Proposition 4.1: A sufficient condition for implication (12) to hold is that at each exploration step k

$$\mathcal{Q}^k \subseteq \bigcup_{j=0}^{k} D(q^j, k). \tag{13}$$

Remark 4.2: Condition (13) requires that, at each step k, the collection of admissible sets $D(q^j, k)$, $j = 0, 1, ..., k$ is a *cover* of the informative safe region $\mathcal{Q}^k$. Obviously, if condition (13) holds, no information is lost and an exploration strategy can *better* select at each step k the next action over the whole range of possible informative actions.

SET METHOD

1: **if** local free boundary $\mathcal{L}(q^k, k)$ is non-empty

2: $\quad (q^{k+1}, U^{k+1}) \leftarrow$ search configuration with maximum utility in $\mathcal{D}(q^k, k) \cap \mathcal{Q}^k$

3: **if** $U^{k+1} > 0$ *%forwarding%*

4: $\quad$ plan a safe path from q^k to q^{k+1}

5: $\quad$ move to q^{k+1} and acquire sensor view

6: $\quad$ update SET and world model

7: **else**

8: $\quad$ **if** $\mathcal{U}^k$ is not empty *%backtracking%*

9: $\quad\quad$ select the closest configuration q^b in $\mathcal{U}^k$

10: $\quad\quad$ plan a path on SET leading to q^b

11: $\quad\quad$ move to q^b

12: $\quad$ **else**

13: $\quad\quad$ homing

Fig. 7. The k-th step of the SET method.

In the next section, we present the SET method and we show how it satisfies condition (13) at each step k (proposition 5.1).

V. THE SET METHOD

In the SET method, the robot incrementally builds the *Sensor-based Exploration Tree* (SET) data structure. Each node of the SET represents a view configuration, while an arc between two nodes is a safe path joining them. A pseudocode description of the k-th step of SET is given in Fig. 7. A comparison with the general exploration method of Fig. 6 already suggests the specific choices that were made. These are detailed in the following.

A. Information Gain

The information gain has been defined in Sect. II-D.

B. Forwarding Selection Strategy

If the condition of line 1 is met (see Sect. V-D), q^{k+1} is selected in $\mathcal{D}(q^k, k) \cap \mathcal{Q}^k$ so as to maximize the utility function $U(q, k) = I(q, k)$ (line 2). A maximum certainly exists because $\mathcal{D}(q^k, k) \cap \mathcal{Q}^k$ is compact and $I(q, k)$ is continuous in q. In principle, the navigation cost from q^k to q^{k+1} could be included in U, to avoid erratic behaviors. However, our definition of $\mathcal{D}(q^k, k)$ together with the adopted search strategy (Sect. VI-A) will give the same result.

C. Admissible Set

To simplify the notation, we assume below that all the sensors have the same perception range R. This does not imply any loss of generality.

Denote by $\mathcal{D}_{r,j}(q, k)$ the *partial admissible set (r,j)* around q at step k defined as the set of configurations w such that *(i)* the r-th sensor center $s_r(w)$ is contained in a ball $B(s_j(q), \rho)$ with radius $\rho \geq R$ and center $s_j(q)$, and *(ii)* $s_r(w)$ and $s_j(q)$ are *mutually visible* at step k, i.e., the open line segment $(s_r(w), s_j(q))$ does not intersect $\partial\mathcal{E}^k$. In this definition, the j-th sensor center $s_j(q)$ acts as a 'fixed pole' while the r-th sensor center $s_r(w)$ can 'move' in $B(s_j(q), \rho)$ as long as it remains visible from $s_j(q)$.

The *admissible set* $\mathcal{D}(q, k)$ around q at step k is defined as:

$$\mathcal{D}(q, k) = \bigcup_{r,j \in \{1,2,\ldots,m\}} \left(\mathcal{D}_{r,j}(q, k) \cap \mathcal{Q}_r^k\right). \tag{14}$$

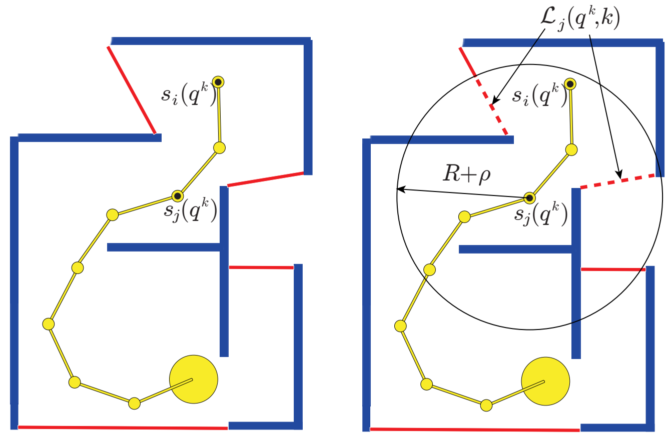

Fig. 8. A reconstructed world model at step k. *Left*: free boundary $\partial\mathcal{E}^k_{\text{free}}$ (red-thin) and obstacle boundary $\partial\mathcal{E}^k_{\text{obs}}$ (blue-thick). The boundary of $\mathcal{E}^k$ is $\partial\mathcal{E}^k = \partial\mathcal{E}^k_{\text{free}} \cup \partial\mathcal{E}^k_{\text{obs}}$. *Right*: the partial local free boundary $\mathcal{L}_j(q^k, k)$ consists in the two dashed segments.

Proposition 5.1: The admissible set (14) is such that:

$$\mathcal{Q}^k = \bigcup_{i=0}^{k} \mathcal{D}(q^i, k) \tag{15}$$

Proof: Let q be a configuration of $\mathcal{Q}^k \neq \emptyset$. Hence, there exists $r \in \{1, 2, ..., m\}$ such that $q \in \mathcal{Q}^k_r \neq \emptyset$. Moreover, the following chain of implications holds

$$q \in \mathcal{Q}^k \Rightarrow \mathcal{A}(q^k) \subset \mathcal{E}^k \Rightarrow s_r(q) \in \mathcal{E}^k \quad .$$

From eq. (2) it follows[6] $s_r(q) \in \mathcal{V}(q^b)$ for some $b \leq k$. In particular, there exists $j \in \{1, 2, ..., m\}$ such that $s_r(q) \in \mathcal{V}_j(q^b)$. Since $\rho \geq R$ then $q \in \mathcal{D}_{r,j}(q^b, k)$. Hence, from eq. (14), it is $q \in \mathcal{D}_{r,j}(q^b, k) \cap \mathcal{Q}^k_r \subset \mathcal{D}(q^b, k)$ and this implies $\mathcal{Q}^k \subseteq \bigcup_{i=0}^k \mathcal{D}(q^i, k)$. Since for any q^i it is $\mathcal{D}(q^i, k) \subseteq \mathcal{Q}^k$, eq. (15) holds. ■

D. Local Free Boundary

The SET method looks at a subset of the free boundary $\partial\mathcal{E}^k_{\text{free}}$ for predicting if $\mathcal{Q}^k \cap \mathcal{D}(q^k, k) = \emptyset$. This results in a significant computational saving, because $\partial\mathcal{E}^k_{\text{free}}$ has dimension $N-1$ whereas $\mathcal{Q}^k \cap \mathcal{D}(q^k, k)$ has dimension n.

Let $\mathcal{L}_j(q, k)$ be the *partial local free boundary* of the j-th sensor around q at step k (Fig. 8), i.e., the set of points of the free boundary $\partial\mathcal{E}^k_{\text{free}}$ that *(i)* are contained in a ball $B(s_j(q), \rho + R)$ with center $s_j(q)$ and radius $\rho + R$, and *(ii)* can be connected to $s_j(q)$ through a world path contained in $\mathcal{E}^k \cap B(s_j(q), \rho + R)$. The parameter ρ of this definition is inherited from the partial admissible sets definition.

The *local free boundary* $\mathcal{L}(q, k)$ is defined as

$$\mathcal{L}(q, k) = \bigcup_{j=1}^{m} \mathcal{L}_j(q, k). \tag{16}$$

$\mathcal{L}(q^k, k) \neq \emptyset$ is a necessary condition for $\mathcal{D}(q^k, k) \cap \mathcal{Q}^k$ to be non-empty, as shown by

Proposition 5.2: The following implication holds:

$$\mathcal{L}(q^k, k) = \emptyset \;\Rightarrow\; \mathcal{D}(q^k, k) \cap \mathcal{Q}^k = \emptyset \tag{17}$$

[6]For ease of presentation, we assume that if $s_r(q) \in \mathcal{A}(q^0)$ then $s_r(q) \in \bigcup_{i=1}^k \mathcal{V}(q^i)$. This assumption may however be removed by suitably defining the admissible set $\mathcal{D}(q^0, k)$ at q^0.

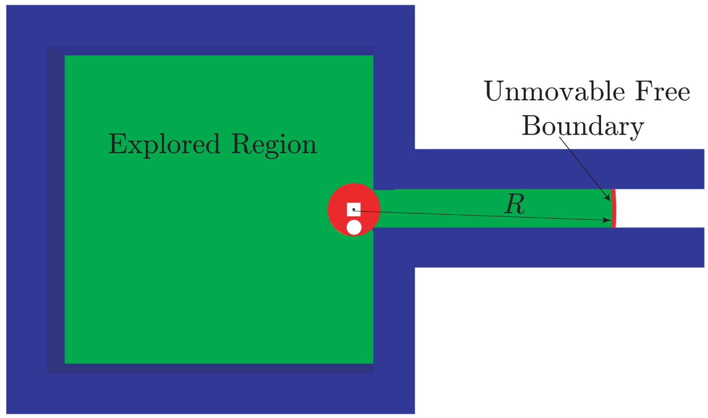

Fig. 9. A planar disc-shaped robot equipped with an omnidirectional range finder (R is its perception range) completes the exploration. The remaining portion of free boundary cannot be pushed forward by any additional sensor view.

Proof: First, one proves that

$$\mathcal{L}_j(q,k) = \emptyset \;\Rightarrow\; \mathcal{D}_{r,j}(q,k) \cap \mathcal{Q}_r^k = \emptyset, \quad r = 1,2,...,m \tag{18}$$

i.e., if $\mathcal{L}_j(q,k)$ is empty, no sensor can gain new information by moving its center around $s_j(q)$ in $B(s_j(q),\rho)$. In fact, assume that $\mathcal{L}_j(q,k) = \emptyset$ and there exists a sensor $i \in \{1,2,...,m\}$ such that $\mathcal{D}_{i,j}(q,k) \cap \mathcal{Q}_i^k \neq \emptyset$. Hence, there exists a configuration $q^* \in \mathcal{D}_{i,j}(q,k)$ such that $I_i(q^*,k) \neq \emptyset$. This implies $\mathcal{V}_i(q^*,k) \cap \partial\mathcal{E}^k_{\text{free}} \neq \emptyset$. Hence, there exists a point $p^* \in \partial\mathcal{E}^k_{\text{free}}$ such that p^* and $s_i(q^*)$ are mutually visible at step k. Since $q^* \in \mathcal{D}_{i,j}(q,k)$, the points $s_i(q^*)$ and $s_j(q)$ are also mutually visible. Hence, p^* and $s_j(q)$ can be connected through a world path completely contained in $\mathcal{E}^k$ (the two line segments $p^* s_i(q^*)$ and $s_i(q^*)s_j(q)$). Moreover, since $\|p^* - s_i(q^*)\| \leq R$ and $\|s_i(q^*) - s_j(q)\| \leq \rho$, then (triangle inequality) $p^* \in B(s_j(q), \rho + R)$. Hence, it follows $p^* \in \mathcal{L}_j(q,k) \neq \emptyset$. This contradicts the starting assumption $\mathcal{L}_j(q,k) = \emptyset$, and therefore eq (18) must hold. At this point, eqs. (14) and (16) are used to obtain the thesis. ■

If $\mathcal{L}(q^k,k) \neq \emptyset$ a search for a new view configuration is attempted in $\mathcal{D}(q^k,k) \cap \mathcal{Q}^k$ (lines 1–2); otherwise no search is performed, U^{k+1} remains zero and the utility check (line 3) is negative. Indeed, even when $\mathcal{L}(q^k,k) \neq \emptyset$, it may happen that $\mathcal{D}(q^k,k) \cap \mathcal{Q}^k = \emptyset$. In general, this occurs when portions of the free boundary cannot be 'pushed-forward' by additional sensor views (e.g., consider the simple case of the planar robot depicted in Fig. 9). Such portions of free boundary belong to the unmovable free boundary.

The *unmovable free boundary at step* k collects all the points $p \in \partial\mathcal{E}^k_{\text{free}}$ for which there is no configuration $q \in \mathcal{R}^k$ such that $p \in \mathring{\mathcal{V}}(q,k)$ (where $\mathring{\mathcal{V}}(q,k)$ is the interior of the simulated view $\mathcal{V}(q,k)$). The unmovable free boundary is due to *(i)* robot kinematic constraints *(ii)* geometric constraints (obstacles in the current environment model) *(iii)* sensory limitations (limited field of view of the sensory system).

The exploration is completed at step k if the free boundary $\partial\mathcal{E}^k_{\text{free}}$ is entirely unmovable. In fact, this clearly implies $\mathcal{Q}^k = \emptyset$. In particular, if the Local Free Boundary $\mathcal{L}(q^k,k)$ is entirely contained in the unmovable free boundary, then $\mathcal{D}(q^k,k) \cap \mathcal{Q}^k$ is empty.

E. Backtracking Selection Strategy

When the search of line 2 returns $U^{k+1} = 0$, the set $\mathcal{D}(q^k,k) \cap \mathcal{Q}^k$ is empty and a backtracking from q^k is attempted (line 7).

```
SEARCH STRATEGY IN D(q^k, k)
q^{k+1} ← 0, U^{k+1} ← 0, J ← {1, 2, ..., m}
while U^{k+1} = 0 and J ≠ ∅
  | j ← select sensor j ∈ J with L_j(q^k, k) ≠ ∅
  | R ← {1, 2, ..., m}
  | while U^{k+1} = 0 and R ≠ ∅
  |   | r ← select by priority sensor r ∈ R
  |   | (q^{k+1}, U^{k+1}) ← search configuration with
  |   | maximum utility in D_{r,j}(q^k, k) ∩ Q_r^k
  |   | if U^{k+1} = 0   R ← R \ {r}
  | end while
  | if U^{k+1} = 0   J ← J \ {j}
end while
return (q^{k+1}, U^{k+1})
```

Fig. 10. A pseudocode description of the search strategy in $D(q^k, k)$.

Let $l(k) \leq k$ be the last exploration step in which a view was acquired. Let $\mathcal{U}^k$ be the set of view configurations q^i such that *(i)* $\mathcal{L}(q^i, k) \neq \emptyset$, and *(ii)* no search has been performed in $\mathcal{D}(q^i, j) \cap \mathcal{Q}^j$ at a step $j > l(k)$ returning $U^{j+1} = 0$.

If $\mathcal{U}^k$ is not empty, the closest view configuration q^b in $\mathcal{U}^k$ is selected as destination (line 9); otherwise, exploration terminates and the robot follows a path on the SET leading back to q^0 (*homing*, line 13).

Proposition 5.3: The following implication holds:

$$\mathcal{U}^k = \emptyset \;\Rightarrow\; \bigcup_{i=0}^{k} \mathcal{D}(q^i, k) \cap \mathcal{Q}^k = \emptyset$$

Proof: If $q_i \notin \mathcal{U}^k$ two cases are possible:

1) $\mathcal{L}(q^i, k) = \emptyset$, hence $\mathcal{D}(q^i, k) \cap \mathcal{Q}^k = \emptyset$ from Prop. 5.2.
2) $\mathcal{L}(q^i, k) \neq \emptyset$ and $\mathcal{D}(q^i, j) \cap \mathcal{Q}^j = \emptyset$ for $j > l(k)$ (since $U^{j+1} = 0$ was returned at a step $j > l(k)$). Given that:
$$\mathcal{E}^{l(k)+1} = \ldots = \mathcal{E}^k, \quad \mathcal{Q}^{l(k)+1} = \ldots = \mathcal{Q}^k$$
it follows that:
$$\mathcal{D}(q^i, j) \cap \mathcal{Q}^j = \mathcal{D}(q^i, k) \cap \mathcal{Q}^k = \emptyset$$

Hence, when $\mathcal{U}^k = \emptyset$, it is $\bigcup_{i=0}^{k} \mathcal{D}(q^i, k) \cap \mathcal{Q}^k = \emptyset$. ■

F. Completeness

Proposition 5.4: Any SET exploration which ends at a finite step k is completed, in the sense that $\mathcal{Q}^k = \emptyset$.

Proof: The SET terminates at step k if $\mathcal{U}^k = \emptyset$, i.e., using Prop. 5.3, if $\bigcup_{i=0}^{k} \mathcal{D}(q^i, k) \cap \mathcal{Q}^k = \emptyset$. Recalling eq. (15), it is $\mathcal{Q}^k = \bigcup_{i=0}^{k} \mathcal{D}(q^i, k) = \bigcup_{i=0}^{k} \mathcal{D}(q^i, k) \cap \mathcal{Q}^k = \emptyset$. ■

The above proposition only considers finite exploration sequences, because a compact free world may not be 'coverable' by a finite sequence of views. In such 'pathological' cases, maximizing $I(q, k)$ over $\mathcal{Q}^k$ results in an infinite sequence of view configurations q^i along which $I(q^i, k)$ tends to zero. Hence, $\mathcal{Q}^k$ never becomes empty.

VI. Implementation

A. Search in the Admissible Set

In general, $D(q^k, k)$ (see Sect. V-C) is a huge search space for the utility maximization problem (line 2, Fig. 7). In order to reduce the search complexity, an heuristic search algorithm can be worked out

SEARCH WITH GLOBAL GROWTH IN $\mathcal{D}_{r,j}(q^k, k) \cap \mathcal{Q}_r^k$

globally expand roadmap $\mathcal{G}^{k-1}$ to obtain $\mathcal{G}^k$

extract from $\mathcal{G}^k$ the subset $\widehat{\mathcal{D}}$ of configurations falling in $\mathcal{D}_{r,j}(q^k, k) \cap \mathcal{Q}_r^k$

find a configuration q^{k+1} of $\widehat{\mathcal{D}}$ with maximum utility U^{k+1}

return (q^{k+1}, U^{k+1})

Fig. 11. A pseudocode description of the SET-GG search strategy.

by relaxing the solution optimality requirement and exploiting the search space inherent decomposition (eq. (14)). This is described in Fig. 10.

Instead of searching for the optimal solution in $\mathcal{D}(q^k, k)$, the algorithm searches one of the *suboptimal solutions* which maximize the utility function in the partial sets $\mathcal{D}_{r,j}(q^k, k) \cap \mathcal{Q}_r^k$, for $r, j \in \{1, 2, ..., m\}$. In particular, the partial sets are visited and searched one by one until the first suboptimal solution is found. The visit order is heuristically designed.

A possible choice is detailed in the following. For any fixed j, implication (18) relates $\mathcal{L}_j(q^k, k)$ to the sets $\mathcal{D}_{r,j}(q^k, k) \cap \mathcal{Q}_r^k$, $r = 1, 2, ..., m$, and, thereby, to their suboptimal solutions. If $\mathcal{L}_j(q^k, k) = \emptyset$, these suboptimal solutions do not exist since $\mathcal{D}_{r,j}(q^k, k) \cap \mathcal{Q}_r^k = \emptyset$, $r = 1, 2, ..., m$; otherwise, a measure of the unexplored points lying in $B(s_j(q^k), \rho + R)$ can be used as an 'optimality indicator' of these suboptimal solutions. Our strategy selects a sensor j with a non-empty $\mathcal{L}_j(q^k, k)$ and with the highest 'optimality indicator' (line 3). Once j has been chosen, the sensor r is selected by priority (line 6). The highest priority is assigned to the index $r \in R$ which minimizes the distance $\|s_r(q^k) - s_j(q^k)\|$. Accordingly, $\mathcal{D}_{j,j}(q^k, k) \cap \mathcal{Q}_j^k$ is the first searched set. Note that it is certainly $q^k \in \mathcal{D}_{j,j}(q^k, k) \neq \emptyset$.

B. Search in Partial Admissible Sets

In the exploration process, SET incrementally updates a model of the configuration space for (i) searching new view configurations and (ii) performing planning operations. Since generic robotic systems typically have high-dimensional configuration spaces, a sampling based approach can be conveniently used to incrementally grow a roadmap which captures the connectivity of the current safe region.

In particular, let $\mathcal{G}^k$ be the roadmap built at step k in the safe region $\mathcal{S}^k$. In $\mathcal{G}^k$, a node represents a safe configuration at step k, while an arc between two nodes represents a local path that is safe at step k and connects the two configurations. Once $\mathcal{E}^k$ is computed merging $\mathcal{V}(q^k)$ with $\mathcal{E}^{k-1}$, the roadmap $\mathcal{G}^k$ is obtained expanding $\mathcal{G}^{k-1}$. During this expansion process, additional sampled configurations which are safe at step k are added to $\mathcal{G}^{k-1}$. In order to find these configurations a collision checking is performed in the reconstructed world model at step k: according to this model, $\mathcal{E}^k$ is the free world and $\partial\mathcal{E}^k$ is the obstacle boundary. In this framework, the SET built at step k represents the path actually traveled by the robot on the roadmap $\mathcal{G}^k$.

Two main instances of the SET method can be obtained depending on the strategy used for growing the roadmap and searching in the partial admissible sets (Fig. 10, line 7). SET with *Global Growth* (SET-GG), which incrementally performs a global expansion of the roadmap $\mathcal{G}^k$. SET with *Local Growth* (SET-LG), which privileges a local expansion of $\mathcal{G}^k$ around the current view configuration q^k.

1) SET-GG Search Strategy (Fig. 11)*:* SET-GG incrementally expands $\mathcal{G}^k$ in the current safe region $\mathcal{S}^k$ using a sampling-based approach such as a multi-query PRM algorithm, or a single-query single-tree algorithm (RRT or EST).

2) SET-LG Search Strategy (Fig. 12)*:* SET-LG first performs a local search around q^k in the attempt to locally maximize the utility function, then, when no local informed configurations are found, it allows a global search (performing possible long jumps). In the local search (Fig. 12, lines 2a–2c): a single-query single-tree algorithm such as RRT or EST is locally expanded. In the global search (Fig. 12, lines 6a–6c): a tree is expanded without performing collision checking (*lazy tree*) inside $\mathcal{D}_{r,j}(q^k, k)$.

SEARCH WITH LOCAL GROWTH IN $\mathcal{D}_{r,j}(q^k, k) \cap \mathcal{Q}_r^k$

1: **if** $q^k \in \mathcal{D}_{r,j}(q^k, k)$
2: *local search* in $\mathcal{D}_{r,j}(q^k, k)$ starting from q^k:
a) expand a tree $\mathcal{T}^k$ rooted at q^k inside a C-space ball with center q^k and radius δ
b) extract from $\mathcal{T}^k$ the subset $\widehat{\mathcal{D}}$ of configurations falling in $\mathcal{D}_{r,j}(q^k, k) \cap \mathcal{Q}_r^k$
c) find q^{k+1} as a configuration in $\widehat{\mathcal{D}}$ with maximum utility U^{k+1}
3: **if** $U^{k+1} > 0$
4: attach $\mathcal{T}^k$ to $\mathcal{G}^{k-1}$ and **return** (q^{k+1}, U^{k+1})
5: **else**
6: *global search* in $\mathcal{D}_{r,j}(q^k, k)$ starting from q^k:
a) expand a lazy tree rooted at q^k inside $\mathcal{D}_{r,j}(q^k, k)$
b) extract from lazy tree a subset $\widehat{\mathcal{D}}$ of configurations q which are safe at step k and such that $I_r(q, k) \neq 0$
c) use a single-query planner to find a configuration q^{k+1} of $\widehat{\mathcal{D}}$ which is reachable from q^k through a path that is safe at step k
d) if no reachable configurations are found in $\widehat{\mathcal{D}}$ then $U^{k+1} \leftarrow 0$
7: **else**
8: *lazy search* for a configuration $q_{r,j}^k$ in $\mathcal{D}_{r,j}(q^k, k)$
9: *global search* in $\mathcal{D}_{r,j}(q^k, k)$ starting from $q_{r,j}^k$: (as above)
10: **return** (q^{k+1}, U^{k+1})

Fig. 12. A pseudocode description of the SET-LG search strategy.

Here, RRTs are preferable for their rapid C-space exploration. In the lazy search (Fig. 12, line 8), a lazy RRT rooted at q^k is expanded (no collision checking) using the following rule: at each iteration, a function that tosses a biased coin determines whether the new generated configuration[7] q_{new} has to be validated before being added to the RRT. If the coin toss yields 'head', q_{new} is added to the tree only if $\|s_r(q_{\text{new}}) - s_j(q^k)\| < \|s_r(q_{\text{near}}) - s_j(q^k)\|$. If the coin toss 'tail', no validation is performed. This expansion proceeds until *(i)* a maximum number of iterations is exceeded, or *(ii)* a configuration $q_{r,j}^k$ is found in $\mathcal{D}_{r,j}(q^k, k)$.

Note that the local search (Fig. 10, line 2), which is first attempted by SET-LG, generates a new view configuration which is distant from q^k at most δ. This mechanism automatically limits the navigation cost of the next robot motion and avoid erratic behaviors. A shortcoming of SET-LG is a non-uniform sampling of the free configuration space. In fact, local searches started at distinct view configurations may expand in overlapping C-space regions. This unwanted result can be almost avoided by suitably selecting the radius δ of the constraining C-space balls (Fig. 10, line 2a).

C. Path Planning

Once a new view configuration q^{k+1} has been selected, a safe path connecting q^k to q^{k+1} is computed by the path-planner. In the SET method, planning depends on the used search strategy. In SET-GG, a safe path is computed on the roadmap $\mathcal{G}^k$. In SET-LG, q^{k+1} is found either by a local search or by a global search. In the first case, a safe path is easily computed on the locally expanded tree $\mathcal{T}^k$. In the second case, the global search strategy automatically returns a path that is safe at step k (Fig. 12, line 6c).

[7] At each RRT iteration, q_{rand}, q_{near} and q_{new} are determined [12].

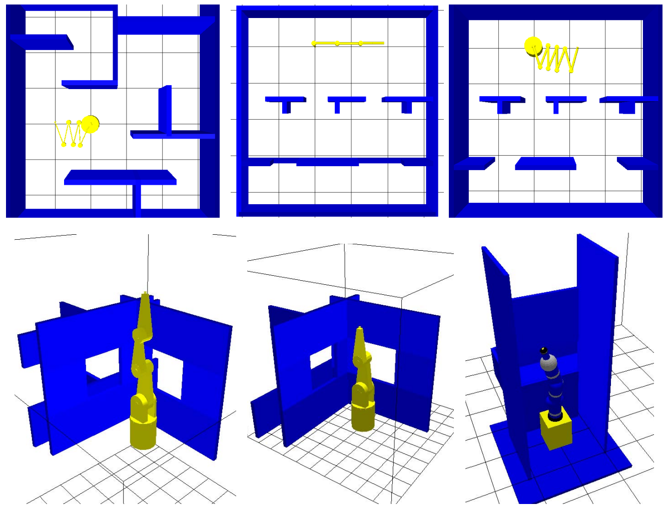

Fig. 13. *Top*: 2D cases (left to right): A6R, B3R*ff*, C8R. *Bottom*: 3D cases (left to right), D4R, E3R, F7R. In each world name, the first letter identifies the scene, while the number quantifies the robot revolute (R) joints; *ff* identifies a free-flying robot.

VII. Simulations

We present simulation results obtained implementing the presented SET method in Move3D [13]. The algorithms have been extensively tested in several scenes (both in 2D and 3D worlds) using various robots (both fixed-base and mobile manipulators). We report here the results obtained in the six cases of Fig. 13. Two groups of simulations were performed for each case: in the first group, a single range finder is mounted on the tip of the robot; in the second, two additional range finders are added and mounted on the last robot link (midway along the length of the link and close to the last robot joint). Each range finder has a perception range $R = 1$ m and an opening angle $\alpha = 60°$ (robot link lengths range from 0.3 m to 0.8 m). Its linear and angular resolution are respectively 0.01 m and 1°. In 2D cases, the sensors can rotate within a $120°$ planar cone; in 3D cases, the sensors can rotate within a $120° \times 120°$ spatial cone. At the start of the exploration, a free box $\widetilde{\mathcal{A}}$ is assumed to be known from an external source. In particular, its volume is 200% of that of $\mathcal{A}(q^0)$ on the average.

Gridmaps are used as world models (with a 0.1m grid resolution). Quadtrees/octrees are used to represent (and efficiently operate on) the free and obstacle boundaries. Information gain is computed via ray-casting procedures. At each step, the partial local free boundary $\mathcal{L}_j(q,k)$ is computed by expanding a numerical 'navigation' function from $s_j(q)$ within $\mathcal{E}^k \cap B(s_j(q), \rho + R)$: any cell in $\mathcal{E}^k \cap B(s_j(q), \rho + R)$ with a finite function value can be connected to $s_j(q)$ and is consequently inserted in $\mathcal{L}_j(q,k)$. Besides, $\mathcal{L}_j(q,k)$ is updated only when $\mathcal{V}(q^{k+1}) \cap B(s_j(q), \rho + R) \neq \emptyset$. Simulations were performed on a Intel Centrino Duo 2x1.8 GHz, 2GB RAM, running Fedora Core 8.

A. Sampling Methods

In SET-GG, the global roadmap $\mathcal{G}^k$ is incrementally expanded using PRM or RRT. In SET-LG, we found that RRT is more effective. In particular, RRT-Extend is used for local searches, while RRT-Connect is more suitable for the lazy tree expansions. In the global searches (Fig. 12, lines 6a–6c), we obtained excellent results by building $\widehat{\mathcal{D}}$ as a single configuration with maximum utility, and then using a bidirectional RRT-ExtCon [12] as a single-query planner to find a path towards the configuration in $\widehat{\mathcal{D}}$.

In all these techniques, kd-trees are used to perform nearest neighbor searches, uniform random sampling is applied and path smoothing is performed.

B. Parameter Choice

- *Admissible set*. The radius ρ in the partial admissible set definition was set to 1 m, i.e, equal to R.
- *RRT*. Each RRT expansion is performed for a maximum number of iterations $K_{\max}$. In SET-LG, a

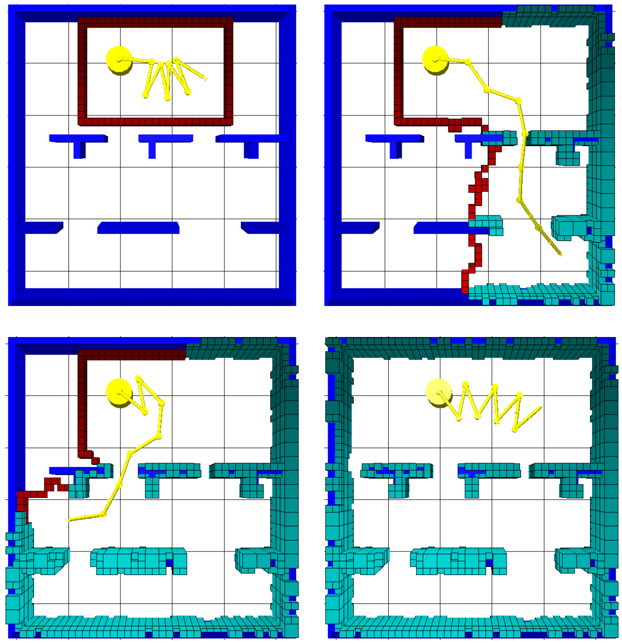

Fig. 14. An exploration progress in world C8R with three range finders.

configuration space ball with radius δ is used in the local search (Fig. 12, line 2a). For both local and global searches, we used $K_{\max} \simeq 2000$, whereas for lazy expansions $K_{\max} \simeq 6000$. As for bidirectional RRT, a higher number of iterations was required ($K_{\max} \simeq 30000$). Typically, we set $\delta \simeq 0.1\,\delta_M$ where δ_M is the maximum estimated distance between two points in $\mathcal{C}$.

C. Performance Indexes

• *Number of Views* (*NV*). It is the total number of views acquired by the robot during an exploration.
• *World Coverage* ($\mathcal{W}_{\%}$). It represents the percentage of the free world included in the final explored region. This percentage is evaluated w.r.t. to an estimate of the free world which can be explored by the robot, i.e., the set of points $p \in \mathcal{W}_{\text{free}}$ such that $p \in \mathcal{V}(q)$ for some configuration $q \in \mathcal{C}_{\text{free}}$ which is reachable from q^0 through a safe path.
• *Number of Collision Detection Calls* (*NCDC*). It is the total number of collision detection calls performed during an exploration.
• *Number of Nodes of the Global Roadmap* (*NNGR*). It is the total number of nodes of the final roadmap $\mathcal{G}^k$.

D. Results

Two typical exploration processes obtained with SET-LG in cases C8R and F7R, both with three range finders, are shown in Figs. 14 and 15. The obstacles (in blue) are obviously unknown to the robot. They are incrementally reconstructed during the exploration as the obstacle boundary $\partial\mathcal{E}^k_{\text{obs}}$ (light-blue cells). In each frame, the free boundary $\partial\mathcal{E}^k_{\text{free}}$ (red cells) is also shown. Fig. 15 shows only the portions of the free boundary contained in the set of points $p \in \mathcal{W}$ such that $p \in \mathcal{A}(q) \cup \mathcal{V}(q)$ for some $q \in \mathcal{C}$. Note that, at the end of the exploration, the remaining free boundary can not be 'pushed-forward' by additional sensor views.

Clips of these two simulations are contained in the video attachment to the paper. Other simulations are available at the webpage http://www.dis.uniroma1.it/labrob/research/SET.html.

Table II compares the results obtained with SET-LG in the case of one range finder and three range finders. In view of the use of RRT, results are averaged over 20 simulation runs. Note that the world coverage is always 100%.

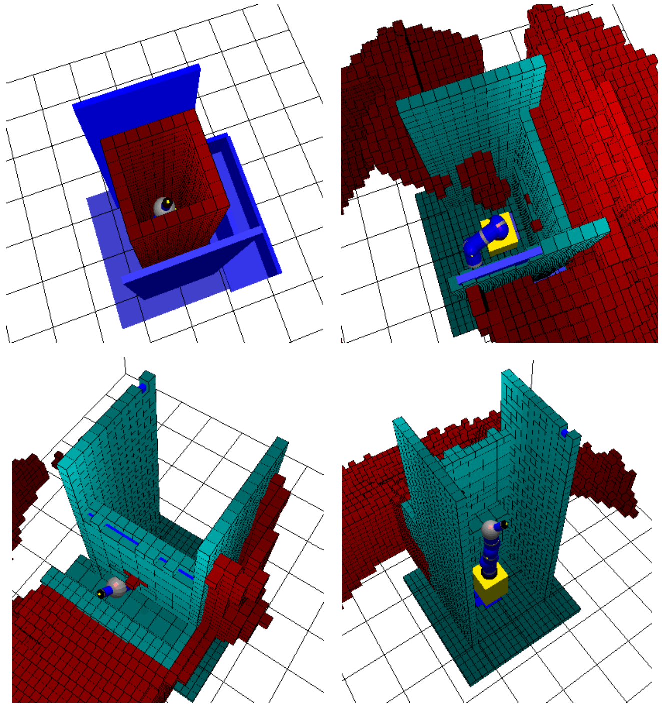

Fig. 15. An exploration progress in world F7R with three range finders.

Results with 1 range finder

World	*NV*	$\mathcal{W}_{\%}$	*NNGR*	*NCDC*
A6R (+1R)	45	100.00%	56938	446314
B3R*ff* (+1R)	48	100.00%	58565	495132
C8R (+1R)	54	100.00%	24662	360591
D4R (+2R)	82	100.00%	48502	323481
E3R (+2R)	79	100.00%	38734	291529
F7R (+2R)	81	100.00%	41679	232913

Results with 3 range finders

World	*NV*	$\mathcal{W}_{\%}$	*NNGR*	*NCDC*
A6R (+3×1R)	30	100.00%	69451	554523
B3Rff (+3×1R)	36	100.00%	86244	615137
C8R (+3×1R)	40	100.00%	52944	432305
D4R (+3×2R)	64	100.00%	77121	671163
E3R (+3×2R)	65	100.00%	84087	571219
F7R (+3×2R)	70	100.00%	91474	598014

TABLE II
RESULTS OBTAINED WITH SET-LG.

E. Comparison of SET-LG with SET-GG

An extensive simulation study has showed that SET-LG performs better than SET-GG. For lack of space, we do not report results obtained with SET-GG. In particular, we found that, for the same maximum number of iterations $K_{\max}$, the world coverage of SET-GG decreases by 10% on the average w.r.t. SET-LG, whereas exploration time increases by 40%.

A comparative analysis of the two methods can justify the results. At each step, the two main computational costs of the SET method are due to: *(i)* the expansion of the roadmap $\mathcal{G}^k$ *(ii)* the extraction of a subset of candidate configurations in $\mathcal{D}(q^k, k)$ from $\mathcal{G}^k$. In particular, at each step, SET-GG expands a global roadmap $\mathcal{G}^k$ which spreads uniformly over the whole safe region as k increases. Clearly, the number of nodes stored in $\mathcal{G}^k$ continuously grows. This causes a parallel, continuous increment of both the above computational costs. On the other hand, at each step, SET-LG mainly expands a new local tree $\mathcal{T}^k$ around the current view configuration q^k. Each of these trees, by construction, has a bounded number of nodes. Hence, with such mechanism, both the described computational costs are in principle bounded and held constant.

Another important advantage of the local growth performed by SET-LG is that it focuses the search process around the current view configuration q^k. This is convenient since, at least in the initial stages of the exploration, new informative configurations are likely to be contained in a neighbourhood of q^k. On the other hand, the global roadmap expansion results in the dispersion of new samples in uninformative configuration-space regions. Also, smaller traveled distance in $\mathcal{C}$ means less energy and exploration time.

VIII. SET in the Presence of Uncertainty

If view sensing comes with uncertainty, a probabilistic world model (e.g. a probabilistic occupancy gridmap [14]) can be used to integrate collected sensor data. Here, a probability distribution associates each representative point in $\mathcal{W}$ with its probability of being in $\mathcal{O}$. Then, a point is classified as *free*, *occupied* or *unknown* comparing its occupancy probability with fixed probability ranges. In this context, SET definitions can be suitably modified. In particular, the explored region (obstacle boundary) is defined as the set of free (occupied) points, a point is unexplored if it is unknown, and the free boundary collects the set of unknown points lying ‘close’ to a free point. All the other definitions accordingly change and an entropy-based measure can be used in the information gain computation.

In a general probabilistic framework, the SET method (with the above modifications) can be thought of as a view planning module which can be suitably integrated with any localization module using a more general definition of utility function U in the spirit of an integrated exploration [2]. Correspondingly, motion planning should be also performed taking into account uncertainty [15].

IX. Conclusion

We have presented a novel method for sensor-based exploration of unknown environments by a general robotic system equipped with multiple range finders. This extends the method originally presented in [11] for single-sensor robotic systems and comes along with a completeness analysis.

The method is based on the incremental generation of a data structure called Sensor-based Exploration Tree (SET). The generation of the next action is driven by information at the world level, where perception process takes place. In particular, the frontiers of the explored region are used to guide the search for informative view configurations. Various exploration strategies may be obtained by instantiating the general SET method with different sampling techniques. Two of these, SET-GG and SET-LG have been described and compared by simulations in non-trivial 2D and 3D worlds.

We are currently working to provide an accurate complexity analysis of the method, improve its completeness analysis and implement the SET method in presence of uncertainty both in sensing and control. Future work will address an experimental validation of SET on a real robotic system, and an extension of the method to a team of robotic systems equipped with multiple sensors along the lines of [5].

References

[1] B. Yamauchi, A. Schultz, and W. Adams, “Mobile robot exploration and map-building with continuous localization,” in *1998 IEEE Int. Conf. on Robotics and Automation*, 1998, pp. 3715–3720.

[2] A. Makarenko, S. B. Williams, F. Bourgault, and H. F. Durrant-Whyte, “An experiment in integrated exploration,” in *2002 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, 2002, pp. 534–539.

[3] H. Gonzalez-Banos and J. Latombe, “Robot navigation for automatic model construction using safe regions,” in *Experimental Robotics VII*, ser. Lecture Notes in Control and Information Sciences, D. Russ and S. Singh, Eds., vol. 271. Springer, 2000, pp. 405–415.

[4] L. Freda and G. Oriolo, “Frontier-based probabilistic strategies for sensor-based exploration,” in *2005 IEEE Int. Conf. on Robotics and Automation*, 2005, pp. 3892–3898.

[5] A. Franchi, L. Freda, G. Oriolo, and M. Vendittelli, “The sensor-based random graph method for cooperative robot exploration,” *to appear in IEEE/AME Trans. on Mechatronics*.

[6] S. Hutchinson, R. Cromwell, and A. Kak, “Planning sensing strategies in a robot work cell with multi-sensor capabilities,” in *1988 IEEE Int. Conf. on Robotics and Automation*, vol. 2, 1988, pp. 1068–1075.
[7] E. Kruse, R. Gutsche, and F. Wahl, “Efficient, iterative, sensor based 3-D map building using rating functions in configuration space,” in *1996 IEEE Int. Conf. on Robotics and Automation*, 1996, pp. 1067–1072.
[8] P. Renton, M. Greenspan, H. ElMaraghy, and H. Zghal, “Plan-n-scan: A robotic system for collision-free autonomous exploration and workspace mapping,” *Journal of Intelligent and Robotic Systems*, vol. 24, no. 28, pp. 207–234, 1999.
[9] P. Wang and K. Gupta, “A configuration space view of view planning,” in *2006 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, 2006, pp. 1291–1297.
[10] W. R. Scott, G. Roth, and J.-F. Rivest, “View planning for automated three-dimensional object reconstruction and inspection,” *ACM Computing Surveys*, vol. 35, pp. 64 – 96, 2003.
[11] L. Freda, G. Oriolo, and F. Vecchioli, “Sensor-based exploration for general robotic systems,” in *2008 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, 2008.
[12] S. M. LaValle and J. J. Kuffner, “Rapidly-exploring random trees: Progress and prospects,” in *Algorithmic and Computational Robotics: New Directions*, B. R. Donald, K. M. Lynch, and D. Rus, Eds. Wellesley, MA: A K Peters, 2001, pp. 293–308.
[13] T. Simeon, J.-P. Laumond, and F. Lamiraux, “Move3d: A generic platform for path planning,” in *4th Int. Symp. on Assembly and Task Planning*, 2001, pp. 25–30.
[14] S. Thrun, W. Burgard, and D. Fox, *Probabilistic Robotics*. Cambridge, MA: MIT Press, 2005.
[15] A. Censi, D. Calisi, A. D. Luca, and G. Oriolo, “A bayesian framework for optimal motion planning with uncertainty,” in *2008 IEEE Int. Conf. on Robotics and Automation*, 2008, pp. 1798 – 1805.